Tutorials in Mathematics

Probability (Intermediate)

This tutorial provides an introduction to the theory of Probability as a rigorous basis for statistics. We will introduce several of the common discrete Random Variables, including Poisson, Bernoulli, Binomial and Geometric, before progressing to continuous Random Variables in the form of the Exponential Distribution, the Normal Distribution and the Uniform Distribution. For each Random Variable, we will explore the concepts of expectation and standard deviation, through the rigorous application of the axioms of probability.

Calculus (Intermediate)

The tutorial presents the tools needed for solving a range of ordinary differential equations (ODEs) and introduces the notion of partial derivatives for use in a variety of applications. For solving ODEs, we will explore methods such as Integrating Factors, Homogeneous Solutions and Separable Solutions. The basics of Partial Differential Equations (PDEs) will also be introduced, with Taylor's Theorem, Fourier Series, and the Gradient Vector.

Linear Algebra (Intermediate)

This tutorial introduces the general notion of a vector space, exemplified through matrices and a generalization of the common two-dimensional and three-dimensional vector systems. After stating the vector space axioms, we will explore properties such as subspaces, bases, linear independence, and dimension. We will also look at how matrices can be used to solve systems of linear equations, and the role of determinants and matrix inverses in this process. Some of the major theorems in the field will be stated and proved, such as the Spectral Theorem and its relationship to eigenvalues and eigenvectors, the Rank-Nullity Theorem, the Dimension Formula, and the Gram-Schmidt Process. Finally, the concept of Linear Transformations will be introduced as a method of mapping between vector spaces.

Abstract Algebra (Intermediate)

Students taking this tutorial will be introduced to the field of abstract algebra through the study of the simplest structure: Groups. We begin with the axioms and the concepts of subgroups and cyclic groups, exemplified by permutations. We then progress to Lagrange's Theorem and Modular Arithmetic, alongside a brief diversion into Number Theory via Fermat's Little Theorem and the Euler Totient Function. The second half of the tutorial focuses on Group Actions and major theorems such as the First and Second Isomorphism Theorems, the Orbit-Stabilizer Theorem, and Burnside's Lemma.

Statistics (intermediate)

The main focus of this tutorial is the theory of Maximum Likelihood Estimation as a means for accurate representation of sample statistics. This includes Hypothesis Testing, Confidence Intervals, and the Central Limit Theorem. The second half of the tutorial extends into Data Analysis via the theory of Principal Component Analysis and the generalization of the statistical concepts explored earlier in the course to higher dimensions.

Complex Analysis (Advanced)

The tutorial begins with a revision of the techniques of real analysis, redefined in the two-dimensional setting of the complex plane. We then move on to define complex differentiation and the concept of a holomorphic function via the Cauchy-Riemann Equations. After a proof of Cauchy's Theorem, the plethora of tools of complex variables become available, such as Cauchy's Integral Formula, Liouville's Theorem, Laurent's Theorem, Residue Theorem, and Contour Integration. If we have time, we will also explore Conformal Maps and their application to solving PDEs.

Differential Equations (Advanced)

The tutorial is split into four main sections: Picard's Theorem, Phase-Planes, Characteristics, and Normal Forms. The first section focuses on establishing the existence and uniqueness of solutions to a given ordinary differential equation (ODE) via Picard's Theorem. Next, we explore coupled systems of ODEs via phase-plane diagrams and the use of nullclines. In the third section we use the method of characteristics to solve some simple Partial Differential Equations (PDEs), before finally introducing Normal Forms and canonical solutions in the fourth section.